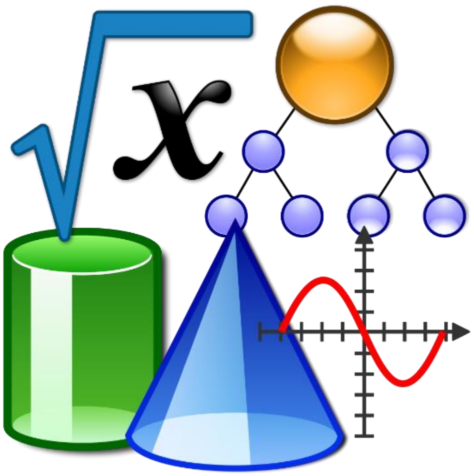
NPS Learning in Place

AFDA



Name _____________School __________ Teacher_________

Week 1	**Probability Review** **Name the Event** **Probability Events** **Venn Diagrams**
Week 2	**Venn Diagrams** **Conditional Probability** **Law of Large Numbers** **Journal/Writing Prompts (Complete at least 2)**

Probability Review

Although probability is most often associated with games of chance, probability is used today in a wide range of areas, including insurance, opinion polls, elections, genetics, weather forecasting, medicine, and industrial quality control.

You have studied probability before in math. Let's see what you remember.

Review Vocabulary: Fill in the correct vocabulary word from the list below

_________________– an occurrence of something, like rolling a six on a single die

_________________– the chances of an event occurring

________________ _______________– collection of all possible outcomes

__________________ *Probability* – determined by mathematical examination of all possible values

__________________ *Probability* – determined by the relative frequencies determined in an experiment

Word Bank

Theoretical **Event** **Experimental** **Probability** **Sample Space**

Probability Review Examples:

Using a six-sided dice, answer the following:

a) P(rolling a six) Answer: 1/6

b) P(rolling an even number) Answer: 3/6 or 1/2

b) P(rolling 1 or 2) Answer: 2/6 or 1/3

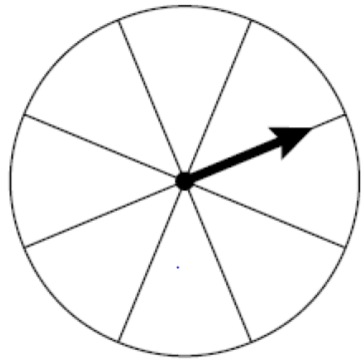
d) P(rolling an odd number) Answer: 3/6 or ½

Review Practice: Solve these problem

1. **Kyle rolls an eight sided dice; what is the probability that he rolls a "3"?**
2. **What is the probability that he rolls a nonprime number (4, 6, 8)?**
3. In a bag of candies there are 13 red candies, 13 green candies, 13 yellow candies, and 13 blue candies. If you choose 1 candy from the bag, what is the probability the candy will *not* be blue?

A) $\frac{1}{4}$ B) $\frac{3}{4}$ C) $\frac{1}{2}$ D) $\frac{2}{3}$

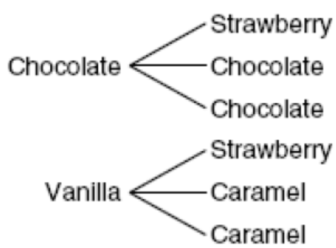
4. How many of the sections of the spinner shown above should be colored blue in order to make the probability of the arrow landing on blue 0.375 in a single spin?



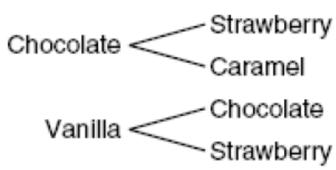
A) 1 B) 7 C) 3 D) 5

5. An ice-cream parlor makes sundaes with chocolate or vanilla ice cream and strawberry, chocolate, or caramel topping. Which tree diagram shows all possible combinations of sundaes with one flavor of ice cream and one topping?

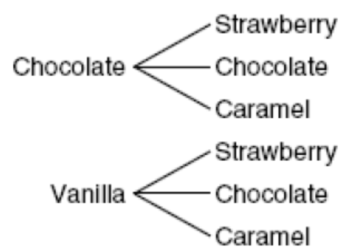
A)



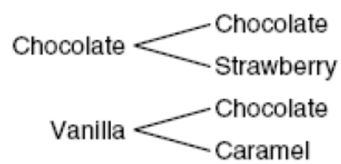
B)



C)



D)



6. The rosebushes in Pattie's Plant Store have the following colors:

- 20% yellow blooms
- 50% red blooms
- 30% pink blooms

If George selected a rosebush at random today, what is the probability that it will produce yellow blooms?

A) $\frac{1}{5}$ B) $\frac{3}{10}$ C) $\frac{4}{5}$ D) $\frac{1}{2}$

7. Joan and Barry are candidates for class president. Orville, Sally, Consuela, Harry, and Rebecca are candidates for vice president. Sam, William, and Frederica are candidates for secretary. How many different combinations of president, vice president, and secretary are possible?

A) 40 B) 11 C) 18 D) 30

Name the Event

Vocabulary

Independent – events are independent if the occurrence of either event does not affect the probability of the other occurring

Dependent – events are dependent if the occurrence of either event affects the probability of the other occurring

Mutually Exclusive – two events are mutually exclusive if the both cannot occur at the same time

Compliment – all outcomes that are not the event

Probability: Complement

Complement of an Event: All outcomes that are **NOT** the event.

When the event is **Heads**, the complement is **Tails**

When the event is **{Monday, Wednesday}** the complement is **{Tuesday, Thursday, Friday, Saturday, Sunday}**

When the event is **{Hearts}** the complement is **{Spades, Clubs, Diamonds, Jokers}**

So the Complement of an event is all the **other** outcomes (**not** the ones we want).

And together the Event and its Complement make all possible outcomes.

Given the following events, identify each as *dependent*, *independent*, or *mutually exclusive*. If they are *mutually exclusive*, say whether or not they are *complementary*.

1. A National Football Conference team wins the Super Bowl and an American Football Conference team wins the Super Bowl.
2. The Washington Redskins win the Super Bowl and the Washington Wizards win the National Basketball Association championship.
3. The Washington Redskins win the Super Bowl and Washington, D.C., has a parade.
4. The first two numbers of your Pick 3 lottery ticket match the winning numbers and the third number does not match.
5. Gus takes the bus to school and he receives a speeding ticket on his way to school.
6. Picking an even number and a number divisible by 5.
7. Joe is flipping a coin 10 times and gets heads each time.
8. Alex is a waiter at Olive Garden and he is a chef at Olive Garden.
9. Herman wants to pick out a pair of black socks from his drawer containing five black pairs, four red pairs and two blue pairs. He picks out a red pair, throws them back in the drawer, and picks out a red pair.
10. Rolling a 4 on a single, six-sided die and rolling a 1 on a second roll of the die.

Probability Events

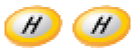
Independent Examples:

- Events are independent if the occurrence of either event does not affect the occurrence of the other
  - P(A and B) = P(A) × P(B)

Example: Probability of 3 Heads in a Row

For each toss of a coin a Head has a probability of 0.5:

0.5



$0.5 \times 0.5 = 0.25$ (or $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$)

$0.5 \times 0.5 \times 0.5 = 0.125$ (or $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$)

And so the chance of getting 3 Heads in a row is **0.125**

You try: What is the probability of rolling 2 consecutive sixes on a ten-sided die?

Dependent Examples:

Dependent Events

But some events can be "dependent" ... which means they **can be affected by previous events.**

Example: Drawing 2 Cards from a Deck

After taking one card from the deck there are **less cards** available, so the probabilities change!

Let's look at the chances of getting a King.

For the 1st card the chance of drawing a King is 4 out of 52

But for the 2nd card:

- If the 1st card was a King, then the 2nd card is **less** likely to be a King, as only 3 of the 51 cards left are Kings.
- If the 1st card was **not** a King, then the 2nd card is slightly **more** likely to be a King, as 4 of the 51 cards left are King.

This is because we are **removing cards** from the deck.

Replacement: When we put each card **back** after drawing it the chances don't change, as the events are **independent**.

Without Replacement: The chances will change, and the events are **dependent**.

- Events are dependent if the occurrence of either event does affect the occurrence of the other
  - P(A and B) = P(A) × P(B|A) **Conditional Probability

You try: What is the probability of drawing 3 aces from a standard 52-card deck?

Mutually Exclusive Examples:

- Mutually exclusive events can't happen at same time
  - P(A or B) = P(A) + P(B) (P(A and B) = 0)

P(A or B) = P(A) + P(B)

*"The probability of A **or** B equals the probability of A **plus** the probability of B"*

Example: King OR Queen

In a Deck of 52 Cards:

- the probability of a King is 1/13, so **P(King)=1/13**
- the probability of a Queen is also 1/13, so **P(Queen)=1/13**

When we combine those two Events:

- The probability of a King **or** a Queen is (1/13) + (1/13) = **2/13**

Which is written like this:

P(King or Queen) = (1/13) + (1/13) = 2/13

You try: What is the probability of rolling a 6 or an odd-number on a six-sided die?	You try: What is the probability of drawing a king, queen or jack (a face card) from a standard 52-card deck?

General Addition Examples:

- Events not mutually exclusive
  - P(A or B) = P(A) + P(B) – P(A and B)

What is the probability of rolling a 5 or an odd-number on a six-sided die?	What is the probability of drawing a red card or a face card from a standard 52-card deck?

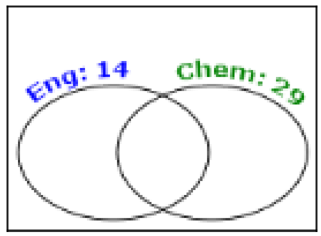
Venn Diagrams and Probability

Venn diagram word problems generally give you two or three classifications and a bunch of numbers. You then have to use the given information to populate the diagram and figure out the remaining information. For instance:

Example: Out of forty students, 14 are taking English Composition and 29 are taking Chemistry.

a. If five students are in both classes, how many students are in neither class?
b. How many are in either class?
c. What is the probability that a randomly-chosen student from this group is taking only the Chemistry class?

Venn



Five students are taking both classes, so put "5" in the overlap.

This leaves nine students taking English but not Chemistry, so put "9" in the "English only" part of the "English" circle.

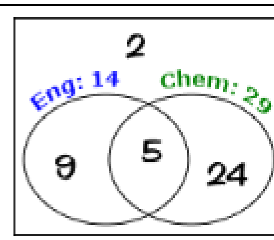
This also accounts for five of the 29 Chemistry students, leaving 24 students taking Chemistry but not English, so put "24" in the "Chemistry only" part of the "Chemistry" circle.

This shows that a total of 9 + 5 + 24 = 38 students are in either English or Chemistry (or both). This also leaves two students unaccounted for, so they must be the ones taking neither class, which is the answer to part (a) of this exercise. Put "2" inside the box, but outside the two circles.

The last part of this exercise asks for the probability that a given student is taking Chemistry but not English. Out of the forty students, 24 are taking Chemistry but not English, which is a probability of:

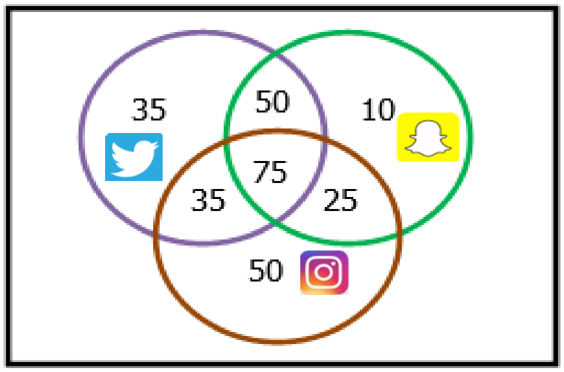
2440=0.6=60%\frac{24}{40} = 0.6 = 60\%4024=0.6=60%

Answers:



a. Two students are taking neither class.
b. There are 38 students in at least one of the classes.
c. There is a 60% probability that a randomly-chosen student in this group is taking Chemistry but not English.

Look at this Venn diagram. What do you notice?



Read the data that the Venn is based on:

A survey was conducted among 300 students regarding the top three social media platforms they are using. The results shows that 195 are on Twitter, 160 are on Snapchat, and 185 are on Instagram. Additionally, 75 are using all three: Twitter, Snapchat, and Instagram. Both Twitter and Snapchat are being used by 125 students; Both Snapchat and Instagram are being used by 100 students. Last, 110 students are using both Twitter and Instagram.

You can calculate probabilities using information provided in the Venn diagram using the following examples. Notice the difference between the use of "and" and "or."

Try these on another sheet of paper:

If a student is chosen at random,

a. What is the probability that the student is using Snapchat or Twitter?
b. What is the probability that the student is using Snapchat and Twitter?
c. What is the probability that the student is not using any of the social media platform?
d. What is the probability that the student is using at least one of the social media platform?
e. What is the probability that the student is using all three social media platforms?

You can find the **conditional probability** with a reduced sample space through the following examples.

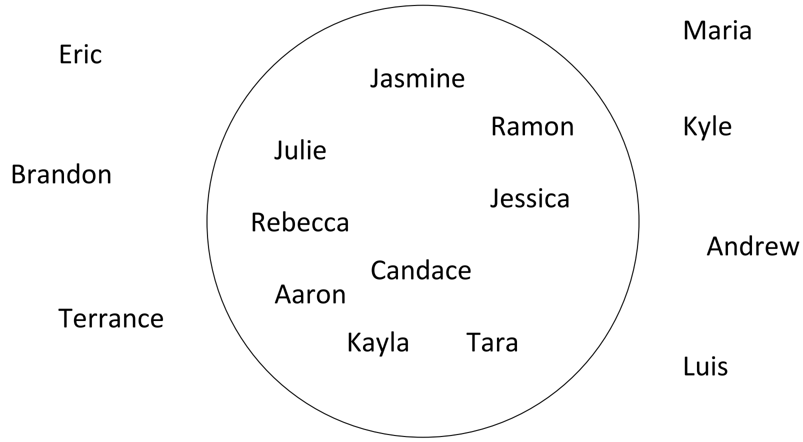
Given that a student uses Twitter,

f. What is the probability that the student is also using Snapchat?
g. What is the probability that the student is also using Instagram?
h. What is the probability that the student is using Snapchat and Instagram?

Venn Diagrams and Probabilities

The Venn diagram below will be used with questions 1-14.

Students with an Earring



Answer the following questions using the Venn diagram above. Use a separate sheet of paper if needed.

1. How many students are in the class?

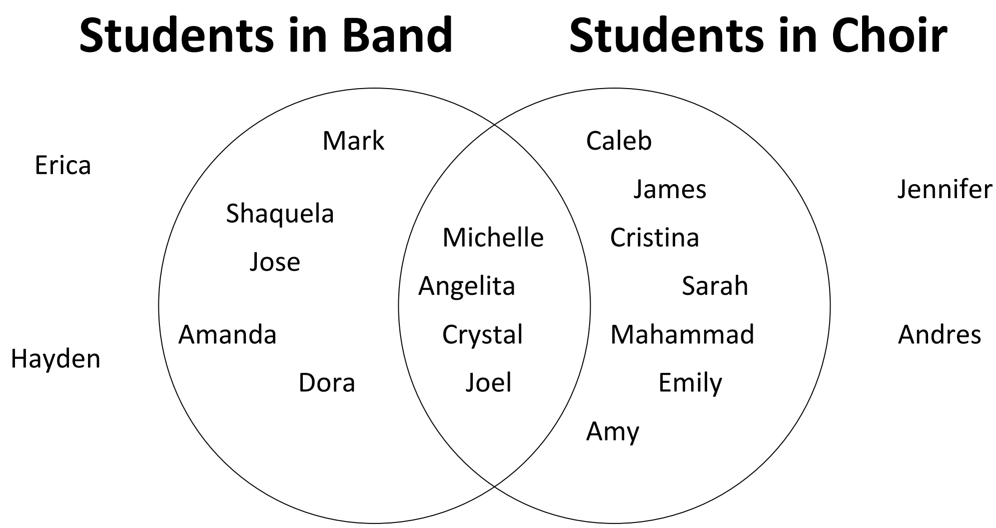
2. How many students have earrings?

3. How many students do not have earrings?

If one student is picked at random, find the following probabilities.

4. P(girl)
5. P(not a girl)
6. P(boy)
7. P(student with earring)
8. P(student without earring)
9. P(boy with earring)
10. P(girl without earring)
11. P(girl with earring)
12. P(boy with earring $\cup$ girl with name starting with J)
13. P(name starting with A $\cup$ name starting with T)
14. Write a probability question which has an answer of $\frac{5}{8}$.

The Venn diagram below will be used to answer questions 15-28. Use a separate sheet of paper if needed.



15. How many students are in the sample?

16. How many students are only in Band?

17. How many students are in Band and in Choir?

If one student is picked at random, find the following probabilities.

18. P(student in choir)

19. P(student in neither band nor choir)

20. P(boy in band)

21. P(girl in band and choir)

22. P(boy in band or choir)

23. P(name starts with C and in choir)

24. P(student in band | boy)

25. P(girl | student in band)

26. P(boy | student in both band and choir)

27. Write a probability question which has an answer of $\frac{1}{2}$.

28. Write a probability question which has an answer of $\frac{4}{7}$.

Conditional Probability

Given that some event has already occurred, what is the probability that another event could occur?

Conditional probability, P(A|B), is read probability of A, given B has already occurred

Conditional probability formula:

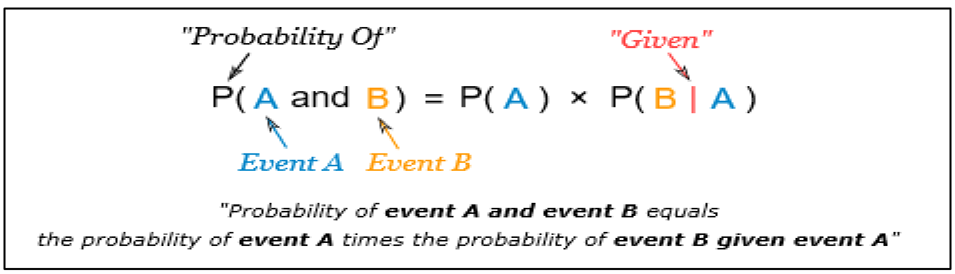
$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

Example:

Given the following information: P(A) = 0.75, P(B) = 0.6 and P(A and B) = 0.33

What is the P(A | B)? $$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{0.33}{0.60} = 0.556$$

What is the P(B | A)? $$P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)} = \frac{0.33}{0.75} = 0.444$$



For contingency tables, probabilities are the number of occurrences listed in the table.

Given information about the number of occurrences of events in tables (experimental probability), we can calculate probabilities (including conditional probabilities) using the following formula:

$$P(A \mid B) = \frac{n(A \text{ and } B)}{n(B)}$$

where

n(A and B) represents the number of observations of both A and B; and
n(B) represents the total number of observations of B

Example:

	Male	Female	Total
Involved in Sports	52	58	110
Not involved in Sports	32	58	90
Total	84	116	200

1. What is the probability of left-handed given that it is a male?
   Answer: P(LH | M) = 12/60 = 0.20
2. What is the probability of female given that they were right-handed?
   Answer: P(F| RH) = 42/90 = 0.467
3. What is the probability of being left-handed?
   Answer: P(LH) = 20/110 = 0.182

Now you try!

	Male	Female	Total
Right handed	48	42	90
Left handed	12	8	20
Total	60	50	110

Solve:

1. What is the probability of being involved in sports given that it is a male?

2. What is the probability of female given that they were involved in sports?

3. What is the probability of not being involved in sports?

Law of Large Numbers

As a procedure is repeated, the relative frequency probability of an event tends to approach the actual probability.

You can create and use your own spinner or use the one on https://www.nctm.org/adjustablespinner/ for this activity.

Law of Large Numbers Activity

Use a spinner (simulator on a calculator or computer). Set the number of sections on the spinner to 5. Change one of the actual (theoretical) values to 50 percent.

1. What has to be true of the other four actual (theoretical) values?

2. What happened to the appearance of the spinner?

Select any five probability values that you would like. Record these actual (theoretical) probability values as percentages.

Section:	1	2	3	4	5
3. Theoretical Values	______	______	______	______	______

4. Record the experimental probability (relative frequency) as a percentage.

After 1 spin	______	______	______	______	______
After 2 spins	______	______	______	______	______
After 5 spins	______	______	______	______	______
After 10 spins	______	______	______	______	______

Change the trial set or number of spins to 10.

After 20 spins	______	______	______	______	______

5. Compare the relative frequency (experimental) probabilities in step 4 with the actual (theoretical) probabilities from step 3. Are you surprised by your results? Why or why not?

6. How many spins do you think it will take for the two types of probabilities to be equal? Explain.

7. Record the probability from relative frequency.

After 30 spins ______ ______ ______ ______ ______

After 40 spins ______ ______ ______ ______ ______

After 50 spins ______ ______ ______ ______ ______

After 100 spins ______ ______ ______ ______ ______

8. Change the trial set or number of spins to 100. Record.

After 200 spins ______ ______ ______ ______ ______

After 300 spins ______ ______ ______ ______ ______

After 400 spins ______ ______ ______ ______ ______

After 500 spins ______ ______ ______ ______ ______

9. Change the trial set or number of spins to 500.

After 1,000 spins ______ ______ ______ ______ ______

After 2,000 spins ______ ______ ______ ______ ______

After 3,000 spins ______ ______ ______ ______ ______

10. Round each of your relative frequency (experimental) probabilities from 3,000 spins to the nearest whole percentage.

______ ______ ______ ______ ______

11. Copy your actual (theoretical) probabilities from step 3.

______ ______ ______ ______ ______

12. Compare the relative frequency (experimental) probabilities in step 10 with the actual (theoretical) probabilities from step 11.

13. Call a classmate! Look at their results. What conclusion can you make about the relationship between relative frequency probability and actual probability as the number of experiments increases?

Journal/writing prompts

Choose at least three of the following prompts to respond to on another sheet of paper. Use correct grammar and mechanics. Use appropriate math vocabulary. Feel free to complete more than three!

Can an event be both mutually exclusive and independent? Why or why not?	Explain, using examples, the difference between conditional probability for dependent events and conditional probability for independent events.	Describe how to find the complementary event for a given event.
Determine the percentage of students who are male compared to female in your school. Explain why this percentage may differ from 50-50.	What are some reasons why the theoretical and experimental probabilities may differ from each other?	Write about a practical situation in which Venn diagrams can be useful to organize the interaction of events. Then, have students explain how probabilities are determined and their usefulness in the practical situation.
Explain the relationship between experimental and theoretical probabilities.	Complementary events are mutually exclusive, but mutually exclusive events are not necessarily complementary. Give an example of events that are mutually exclusive but not complementary and describe why they are not complementary.	Discuss how theoretical probabilities can be used to estimate the actual probabilities of a large data set. Explain the pros and cons of using the Law of Large Numbers.